Exact small ball asymptotics in Hilbert norm for Gaussian processes

The theory of small deviations (sometimes called the theory of small balls) for Gaussian processes has developed rapidly over the recent years. Such a development was stimulated by numerous links between this theory and some important mathematical problems, including the accuracy of discrete approximation for random processes, the calculation of the metric entropy for functional sets, the law of the iterated logarithm in the Chung form, the quantization problem, the Bayesian estimation, andthe functional data analysis.

Our survey describes the state of art for the theory of small deviations in L_2 norm. The main idea is to use the spectral asymptotics for the integral operators defined by the covariances of Gaussian processes under consideration. We obtain the exact small deviation asymptotics in the final form for many known Gaussian processes and extend our results to the Brownian excursion, Brownian meander, and Bessel processes and bridges.

Tests of fit based on characterizations, and their efficiencies

Goodness-of-fit testing has always been very important area of Statistics. We present a survey of relatively new direction of research connected with tests of fit based on characterizations of distributions. The test statistics are functionals of so-called U-empirical processes. Most explored are integral statistics and supremum statistics of Kolmogorov type.

We describe their limit theory and large deviation asymptotics under the null hypothesis. Next their local Bahadur efficiency for parametric alternatives is calculated. This type of efficiency is mostly appropriate here since the Kolmogorov type statistic is not asymptotically normal, and the Pitman approach is not applicable. The conditions of local optimality for new statistics in the Bahadur sense are also discussed. Some new directions of research are suggested.

U-max statistics and limit theorems for random polygons

Recently W. Lao and M. Mayer in Germany and in Switzerland considered“U-max – statistics”, where instead of the sum appears the maximum over the same set of indices. Such statistics often appear in stochastic geometry. The examples are given by the largest distance between random points in a ball, the maximal diameter of a random polygon, the largest scalar product within a sample of points, etc. Their limiting distributions are related to the distributions of extreme values.

Lao and Mayer obtained limit theorems for the maximal perimeter and the maximal area of random triangles inscribed in a circumference. We generalize their theorems to convex m-polygons, m > 3, with random vertices on the circumference. Next, a similar problem is solved for the minimal perimeter and the minimal area of circumscribed m-polygons which has not been previously considered in literature. We also partially generalize our results for the ellipse. Finally, we discuss the obtained results when m grows.