Study programmes: Bachelor studies - Mathematics

Course name: Probability Theory

Lecturers: Pavle N. Mladenović

Status: Compulsory for the module Statistics, actuarial and financial mathematics

ECTS: 10

Attendance prerequisites: Introduction to probability

Course aims: Acquiring general and specific knowledge from probability theory, based on measure theory. Construction of typical probabilistic models of random experiments. Learning concepts of convergence and limit theorems in sequences of independent random variables.

Course outcome: Upon completing the course a student has basic knowledge in probability theory and is capable to learn all special topics from other subjects on the module Statistics, actuarial and financial mathematics.

Course content:

σ-algebra. Measurable space. Probability space. Kolmogorov's axioms. Basic properties of probability. discrete probability space. Conditional probability and Independent events. Borel-Cantelli lemma. Monotone sequences and monotone classes. Carathéodory's extension theorem. Complete probability space and Approximation theorem. Distribution function. Correspondence between distribution functions and probability measures on real line. Discrete distributions. Absolutely continuous distributions. Singular distributions. Probability in high dimensions. Multidimensional distributions. Probability on infinite-dimensional spaces (Cylinder sets, Borel sets, Consistency conditions). Kolmogorov extension theorem. Random variable. σ-algebra generated by a random variable or by a family of random variables. Random vectors and Borel functions of random vectors. Independence of random variables. Mathematical expectations. Moments of random variables and Moment inequalities. Covariance and correlation. Covariance matrix of random vectors. Conditional mathematical expectation. Characteristic functions (Definition, Basic properties, Product theorem, Inversion formulae). Continuity of correspondence between characteristic functions and distribution functions. Method of characteristic functions. Multivariate normal distribution. Linear combination of normally distributed random variables. Convergence of sequences of random variables. Relations between different types of convergence. Cauchy's criterion for convergence Skorohod's theorem. Tail events and tail σ-algebra. Kolmogorov's zero–one law. Law of large numbers. Kolmogorov's inequality. Strong law of large numbers. Central limit theorem. Lindeberg-Feller theorem. Lyapunov theorem. Law of the iterated logarithm. Infinitely divisible distributions and central limit problem. Random series. Kolmogorov's two-series theorem. Kolmogorov's three-series theorem.

Literature:

Павле Младеновић: *Вероватноћа и статистика*, Математички факултет, Београд 2008.
А.Н. Ширяев: *Вероятность*, Наука, Москва, 1989.

Number of hours: 8	**Lectures**: 4	**Exercises**: 4

Teaching and learning methods:
Lectures (frontal). Classes and exercises (interactive).

Assessment (maximal 100 points)

Course assignments	**Points**	**Final exam**	**Points**
Lectures		Written exam	
Exercises/Tutorials	10	Oral exam	40
Colloquia	40	Written-oral exam	
Tests	10		